**1. [paper based, 9 points]**

Use polynomial division to obtain the quotient and the remainder for the following pairs of polynomials (i.e. find polynomials Q and R with f = gQ + R).

A) f = x^4 + 3x^3 + 2x^2 + 6x - 5, g = x - 2, coefficients are from the rational numbers

B) f = x^3 + 2x^3 + 3x^2 + 4x + 1, g = x + 2, coefficients are understood modulo 5.

C) f = x^7 + x^5 + x^2 + 1, g = x^2 + x + 1, coefficients are understood modulo 2.

**2. [paper based, 9 = 4 + 4 + 1 points]**

The polynomials below are above the rationals.

A) Describe all values of P and Q for which the polynomial f = x^4 + 3x^3 + Px^2 - x + Q

has remainder 7 when divided by g = x-2.

B) Describe all values of P and Q for which the polynomial f = x^3 + 2x^2 + Px - 2Q

has no remainder when divided by g = x-3.

C) Give the common solutions for A) and B).

**3. [computer+paper based, 3 + 3 + 3 points]**

A) Find the rational polynomial G of degree at most 4 which has the property that G(0) =1, G(1) = 2, G(2) = 4, G(3) = 8 and G(4) = 16. What is G(5) and G(6)? You may use built-in Lagrange interpolation in sage.

B) Find the modulo 7 polynomial G of degree at most 4 which has the property that G(0) =1, G(1) = 2, G(2) = 4, G(3) = 8 and G(4) = 16. What is G(5) and G(6)? You may use built-in Lagrange interpolation in sage.

C) (Reason on paper, but you may use sage to “guess” the answer). Is there a modulo 4 polynomial G for which G(0) =1, G(1) = 2, G(2) = 4, G(3) = 8 and G(4) = 16?

**4. [paper or computer based, 9 points]**

Using brute force search (or paper methods), find (describe) all the values of p, q and m for which x^4 + px + q is a multiple of x^2 + mx + 1 in the ring of modulo 13 polynomials.

**5. [computer based, 9 points]**

A coding method operates on modulo 5 sequences by replacing the original message (a, b, c) by the codeword (a, b, c, a+b+c, a+2b+abc). Compute the distance of the code, which is defined as the minimal distance between two distinct codewords. The distance of two codewords is defined as the number of positions where they differ. Altogether there are 125 codewords corresponding to all original messages {(a,b,c) | a, b, c = 0, 1, 2, 3 or 4}.

Increase the distance by one by removing codewords from the code. Try to remove as few words from the set of 125 codewords as you can.